Study Unit

Consumer Math

Part 1

Preview

You probably know more about math than you think you do. You've been applying mathematics to money matters since you were a child. You calculated the coins needed for a vending machine and counted your change from a store purchase. As you got older, you may have learned to balance your checkbook.

In *Consumer Math, Part 1,* you'll review the math you already know. Then, you'll learn simple ways to apply math to everyday areas of your life—most of them involving money.

If you're unfamiliar with any terms mentioned, refer to the Glossary at the end of the study unit.

When you complete this study unit, you'll be able to

- Solve any math problem using six specific steps
- Understand what a valuable tool basic math can be in everyday situations
- Add long columns of numbers using a quick, easy method
- Estimate results quickly, using the technique of *rounding off*
- Change fractions to percents, and vice versa
- Figure the amount of money involved when discounts are stated in percents

Contents

Consumer Math, Part 1

LEARNING ABOUT MATH AGAIN

John Weston used a credit card to buy $800 worth of furniture. He used another credit card to buy some new clothes for $300, and he used a gasoline credit card to buy four new tires for his car at a cost of $240. At the first of the month, John had to make payments on these purchases, pay the rent, and buy food. He didn't have enough money to do all of these things, and he had to return some of the furniture. Could John have avoided this financial crisis? Yes. If he knew how to apply basic math, he could have budgeted his purchases over a period of time. He would have avoided this crisis and may have had money to spare (Figure 1).

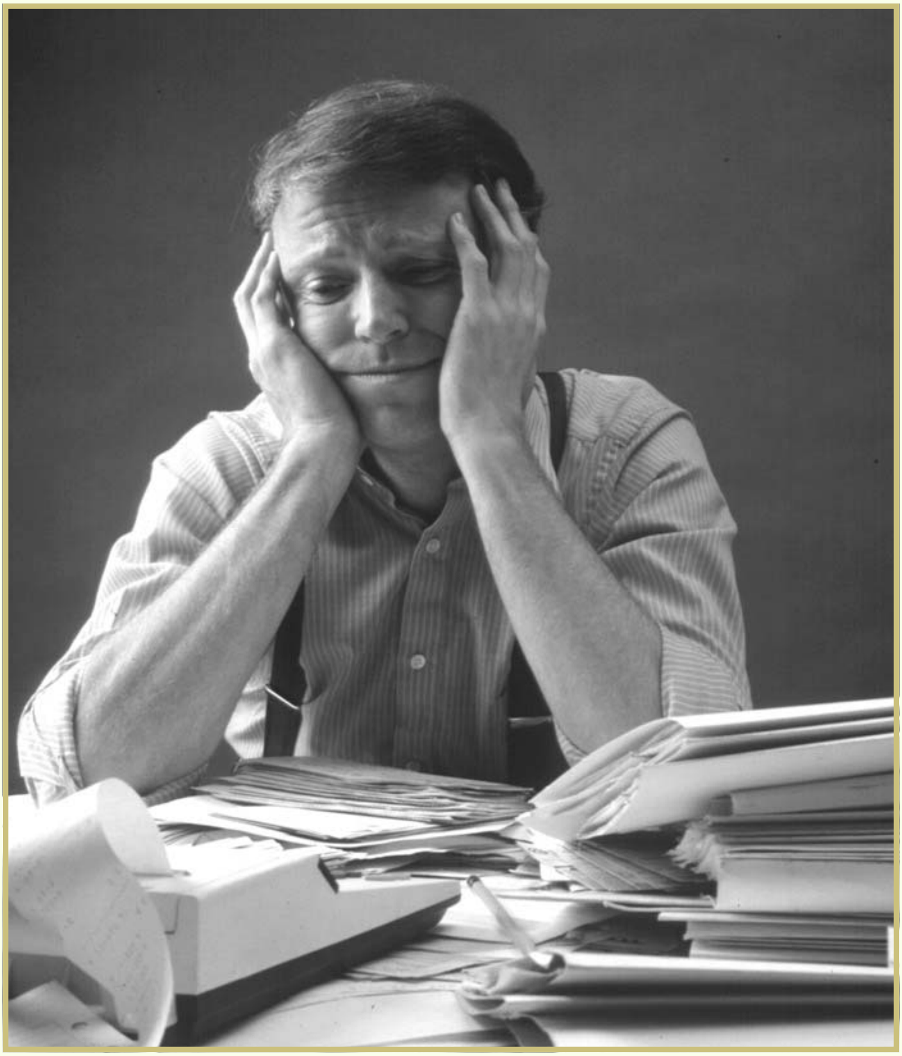


FIGURE 1—Have you ever felt overwhelmed trying to make ends meet?

Undoubtedly, you want to spend and invest your money as wisely as you can. That's what *Consumer Math, Part 1* will help you to do. If you learn and apply basic math principles, you can avoid situations like the one experienced by John Weston. It all begins with understanding mathematics.

You may be thinking, "We're in the computer age now. Computers handle money matters-bank accounts, credit cards, billing systems, and so on. So why should I bother to learn what computers can do so easily and so well?"

It's true that computers do perform well and never make mistakes. But people feeding information to computers *do* make mistakes—*with your money*. So, for your own protection, you should know how to check on the way people and computers handle your money.

To solve these everyday math problems, we'll take some time to review basic math skills.

Six Steps in Problem Solving

Before we begin the review of basic math, let's get into a "think math" frame of mind. To do this, we'll follow six steps that can be used to solve any math problem.

Step 1

Read the problem carefully so that you know what's given and what's required. In other words, don't try to do a problem before you know what it's all about. This is so obvious that you'll probably wonder why it's mentioned. But the downfall of many people who can't solve mathematical problems starts right at this point. Don't begin your computation until you have a clear picture of what's needed. Read and reread the problem, and then read it again if there's the slightest doubt in your mind about its meaning.

Step 2

Write down the facts and figures that are given and required. That is, write down what you know and what you have to find out. Doing this will impress the main points on your mind. It will also let you see whether you understand the given data.

Step 3

Try to find some relation that exists between the given quantities and those you must find. This step requires thinking. That's why it's the most difficult step. There are no rules or definite procedures to help you. You're on your own. Here, your intelligence, resourcefulness, and knowledge will play the major roles.

Step 4

Decide which mathematical operation will produce the desired result. If your analysis of the problem in Step 3 has been clearly made, Step 4 will be merely a matter of putting that analysis into mathematical language.

Step 5

Do the necessary work in a neat and orderly manner and label the result correctly. Neatness and order are important not only to you but also to anyone who looks over your work. Watch someone who is good at problem solving. Observe how carefully this person works and explains the steps. Notice how easy it is to follow the steps when they're done neatly and orderly.

Step 6

Check your work. Use your answer in checking the problem, and be sure that your answer is a reasonable one. Any result is absolutely worthless if it's not correct. Everyone makes mistakes, but the competent person will find any mistakes and correct them.

How to Use the Six Steps

Don't say to yourself, "I won't need to follow these six steps because my friends don't follow them," or "This is too elementary for me." It's true that sometimes the experienced problem solver doesn't visibly perform each step. But you can be sure the person hasn't omitted any steps. Undoubtedly, the person has learned how to do some of the work mentally. You, too, will acquire this skill after you've solved enough problems.

But until you have this experience, you should write down each step (Figure 2). Now let's see how these steps can be used in solving an actual problem.

FIGURE 2—Any mathematical problem can be solved using these six steps.

STEP 1

Read the problem carefully.

STEP 2

Write down the facts and figures.

STEP 3

Find a relation between what's given and what must be found.

STEP 4

Decide which mathematical operation will produce the desired result.

STEP 5

Do the work neatly and label the answer correctly.

STEP 6

Check your work.

Example Problem

Throughout this study unit, you'll find example problems to illustrate each principle, rule, and formula. Read every example carefully. Then, study the solution until you understand it thoroughly. The problem given below illustrates how to apply the six steps of problem solving. Use these six steps whenever you can't readily solve a problem.

Example: An article whose net weight is 10 lb (pounds) is packaged in a box that weighs $\frac{1}{2}$ lb. If 20 of these boxed articles are put into one freight container that weighs 15 lb, what is the gross weight?

Solution: . Think your way through the problem, following the step-by-step method above.

Step 1: I'm given the weight of an article, its individual container, and the freight container that holds 20 of the smaller boxes. I'm required to find the gross weight, which is the total weight of the shipping container and its contents.

Step 2: The quantities are 20 articles weighing 10 lb each; 20 boxes weighing $\frac{1}{2}$ lb each; and a 15-lb freight container.

Step 3: The gross weight is the weight of the 20 articles plus the 20 boxes plus the freight container.

Step 4: To get the gross weight, I'll *add* the weights of the articles, boxes, and the freight container. To get the weight of 20 articles and boxes, I'll *multiply* the weight of each by 20.

Step 5: My calculations are

20×10 lb	= 200 lb
$20 \times \frac{1}{2}$ lb	= 10 lb
Weight of 20 articles and boxes	= 210 lb
Weight of container	= 15 lb
Gross weight	= 225 lb

Answer: The gross weight is 225 lb.

Step 6: The problem asked for gross weight, and I've found it. I can check my work like this:

$10\text{ lb} + \frac{1}{2}\text{ lb} = 10\frac{1}{2}$ lb (Weight of article and box)

$20 \times 10\frac{1}{2}\text{ lb} = 210$ lb (Weight of 20 articles and boxes)

$210 + 15\text{ lb} = 225$ lb (Weight of articles, boxes, and container)
225 lb is the gross weight.
This is the same answer I got before.

Notice that the checking was done by using a method different from the method used in the solution. *It wasn't simply a repetition of the original calculations.* Using another method will often prevent making the same error over again.

Math-A Practical Tool

Now that you're "thinking math," try to imagine a world *without* mathematics! You would have no way to measure temperatures, estimate gas mileage, determine the day of the year, or check the time of day. There would be no way to count items or figure what something is worth. Money wouldn't exist. Nothing would be of standard size, shape, or density. Hardly any of the world's inventions would exist. Human beings would have no reasoning power.

So mathematics has served man well. Today, the average person is constantly calculating and figuring out answers to problems without using an adding machine, calculator, computer, or even a piece of paper and pencil. Most of these quick calculations are done in seconds and require little thought.

When you're driving along the highway and you suddenly become aware that the gas gauge is showing *empty*, your mind quickly calculates where the nearest town and gas station are. Based on your estimate of the distance and how far you've driven the car before when the gauge showed empty, you figure whether or not you can make it to the station.

Or let's say you're in charge of purchasing the food for the company picnic. Easy task? It *is* easy, if you know how many people are attending the event, have an accurate estimate of how much food and drink the average person will consume, and have an unlimited budget. But what happens when management imposes a tight budget? Then, the fun begins! You've got to become creative and figure how to s-t-r-e-t-c-h the dollars so that there will be enough food and beverages. You find yourself calculating and comparing ounces, liters, pieces, pounds, packages, and sizes.

A situation similar to the following one has probably happened to you. You pass the drugstore on your way home from work, when you suddenly remember that you're supposed to pick up some vitamin pills. While in the store, you see a special sale on the brand of toothpaste you use. You quickly estimate the total cost of the vitamins and the toothpaste. You mentally compare this total with the cash in your wallet to see if you can afford the toothpaste, too.

The amazing thing about making computations of this sort-whether figuring gas mileage, purchasing picnic goodies, or replenishing the medicine cabinet-is that all of them involve only four kinds of mathematics: addition, subtraction, multiplication, and division. These four operations are basic to all of the everyday situations you encounter in your life (Figure 3). This course teaches you how to apply simple math to practical living situations, so that you can calculate quickly and accurately. Once you learn these basics, you can improve your efficiency and become a better manager of both your math and your money.

To get the most out of your learning, it's best to put aside all adding machines, calculators, and computers, and use only pencil and paper. Most math becomes easy once you discover the steps involved. And the only way to really understand these steps is to follow them through, from start to finish, in your own mind. After all, you possess a tremendously efficient computer-the human brain-which is capable of performing just about any math task you put before it.

FIGURE 3—Whether purchasing groceries to prepare a family meal, buying a new car, or saving for your children's education, you'll need to know how to add, subtract, multiply, and divide.

Accuracy and Speed

Your ability to compute depends upon two things-accuracy and speed. Accuracy without reasonable speed can't satisfy the demands of today's fast and highly competitive life. Nor can speed without accuracy be of any value to you (Figure 4). First of all, you should acquire accuracy in all of your work. Then, having attained accuracy, try to develop speed. You'll have to honestly determine how much practice you need. The many practice problems included in this study unit will help you evaluate your level of comprehension. Be sure you understand, and can apply, each new idea before you proceed to the next one. Before going on to refresh your addition skills, take time to complete *Count Your Change 1*.

FIGURE 4—In both problem solving and race car driving, speed without accuracy can be costly.

Count Your Change 1

At the end of each section of *Consumer Math, Part 1*, you'll be asked to check your understanding of what you've just read by completing a "Count Your Change." Writing the answers to these questions will help you review what you've learned so far. Please complete *Count Your Change 1* now.

Questions 1–4: Indicate whether the following statements are True or False.

_____ 1. You should write down the facts and figures of any given problem to find out what you know and what you need to determine.

_____ 2. There's usually no relationship between the given quantities of a problem.

_____ 3. Your ability to compute mathematical problems depends first on speed, then on accuracy.

_____ 4. The sixth and final step in mathematical problem solving is checking your work.

Check your answers with those on page 51.

ADDITION

Terms You'll Need to Know

In the operation of addition, the numbers being combined are called *addends.* The answer you get when you add the addends is called the *sum.*

$$\begin{array}{r} 9 \\ 1 \\ 5 \\ 4 \\ 2 \\ 4 \\ 7 \\ 3 \\ \underline{+\ 6} \end{array}$$

The foundation of good addition skills is the ability to give the sum of any two single-digit numbers at a glance. When you're adding long columns of numbers, you should look for figure combinations that add up to 10. For example, when adding the column at the left from the bottom up, think 6, 16, 26, 31, 41. How did we arrive at this series of numbers?

If adding this column of numbers is broken into steps, you'll find you've performed the following mental steps:

$6 + (3 + 7) = 6 + 10 = 16$
$16 + (4 + 2 + 4) = 16 + 10 = 26$
$26 + 5 = 31$
$31 + (1 + 9) = 31 + 10 = 41$

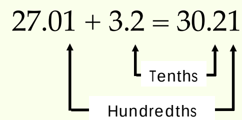
Have you noticed that in most cases you've combined numbers that add up to 10 ($3 + 7$, $4 + 2 + 4$, and $1 + 9$)?

Don't add the column of numbers this way: $6 + 3 = 9 + 7 = 16 + 4 = 20 + 2 = 22 + 4 = 26 + 5 = 31 + 1 = 32 + 9 = 41$. It's much too slow. After a little practice, you'll be able to recognize the combinations totaling 10, regardless of the order in which the numbers appear.

Horizontal and Vertical Addition

Many times you'll be asked to find the sum of numbers that are in a horizontal row instead of in a vertical column. For this reason, you should learn horizontal addition so that you can do it just as easily as vertical addition. *Don't rewrite the numbers* if they're given in a horizontal row. You should train your eye to move to the right or to the left as well as up or down.

In horizontal addition, you must be especially careful to add only those digits having the same place value. Figure 5 illustrates place values. Thus, you'll add only numbers in the ones place to other numbers in the ones place, tens to tens, and so forth. Also, remember that if there's a decimal point in any one of the numbers added, there will be a decimal point in the sum.



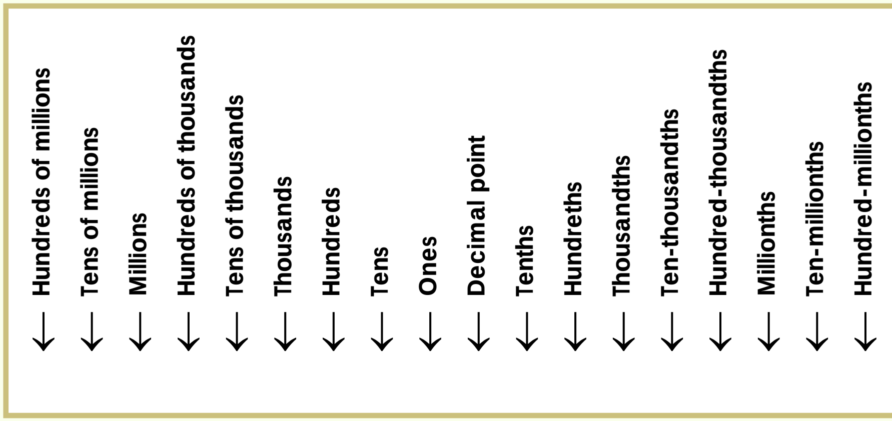


FIGURE 5—This diagram will help you understand the relation of decimals and whole numbers. The name of each place, or position, is given above each digit of the number. Using the ones place as the starting point, you can see that whole numbers increase to the left, while decimals decrease to the right.

Errors regarding place value are less likely to occur when adding numbers in vertical columns, since the decimal point in each number is placed directly below the decimal point of the number preceding it. The decimal point of the sum should be aligned with the decimal point in the addends:

$$\begin{array}{r} 27.01 \\ +\underline{\ \ 3.2\ \ } \\ 30.21 \end{array}$$

How to Check Addition

The best way to check addition is to add the figures again. But don't merely repeat the order you used the first time; add in the *opposite direction*. In other words, if you added *up*, check by adding *down*. If you added from *left to right*, check by adding from *right to left.*

Before going on to refresh your subtraction skills, take time to complete *Count Your Change 2*.

Count Your Change 2

A. Write the sums of the following numbers in the space provided under each problem. Practice until you can give all the sums correctly in three minutes or less.

1.	2	7	0	8	2	0	1	2	4	3
	1	8	1	7	9	6	3	0	5	4
2.	4	6	7	8	9	5	4	7	4	7
	3	7	1	8	2	5	6	0	0	4
3.	2	2	2	3	5	9	9	8	5	1
	7	4	3	8	7	1	3	6	8	1
4.	0	3	3	2	4	1	8	0	6	6
	6	1	6	8	2	7	0	3	6	8
5.	7	4	8	7	3	6	3	4	9	8
	7	7	5	6	5	1	9	4	4	1
6.	3	8	9	4	5	0	5	3	4	2
	0	4	9	1	6	9	0	3	8	5
7.	6	1	9	8	1	6	0	7	7	9
	5	2	5	2	5	0	0	2	5	6
8.	0	6	0	6	4	8	1	1	5	6
	2	9	7	2	9	3	4	8	2	3
9.	9	1	7	2	9	3	0	6	9	1
	8	6	9	6	0	7	8	4	7	0
10.	7	5	1	5	5	0	3	8	5	2
	3	4	9	3	9	4	2	9	1	2

B. Add the following columns of numbers.

1.	2.	3.	4.	5.
13	13	317	1127	13103
35	37	226	2632	61706
12	92	232	1946	43285
26	35	518	3217	39134
73	64	396	1184	96286
24	22	435	1936	45173
40	30	707	2003	78135
89	97	992	5114	26232
23	18	311	976	19375
41	33	417	5634	8428

Check your answers with those on page 51.

SUBTRACTION

Terms You'll Need to Know

There are two methods of subtraction-the *additive* method and the *take-away* method. Both are fairly easy to learn; they're really just the reverse of one another.

In the additive method, you can use the skills you developed in addition. If you were asked to solve 18 - 7 = ?, you could simply ask yourself: 7 + ? = 18.

With the take-away method, you think "18 take away 7 leaves 11." Sometimes it's easiest to picture objects in a group. After counting the number being taken away, count those remaining. You may use this method until you get the hang of it and can work with larger numbers.

The number that you're taking from another is called the *subtrahend*. The number you're taking it from is called the *minuend*. The answer to the subtraction problem is called the *difference*. Don't let these terms intimidate you. They're just names for this:

208	Minuend (the number with which you're starting)
–135	Subtrahend (the number you're subtracting)
73	Difference (the solution)

How to Check Subtraction

It's very easy to see whether your subtraction is correct. Simply add the difference and the subtrahend. It should equal the minuend. If it doesn't, something is wrong. You must then check your work to find the mistake.

Before going on to refresh your multiplication skills, take time to complete *Count Your Change 3*.

Count Your Change 3

Complete the following by subtracting the lower number from the upper number. Practice until you can do this exercise in four minutes or less.

1.	2 − 2	2 − 1	17 − 8	10 − 1	8 − 7	12 − 9	10 − 5	11 − 3	2 − 0	14 − 5
2.	13 − 4	4 − 3	16 − 7	7 − 1	8 − 8	9 − 2	5 − 5	14 − 6	7 − 0	4 − 0
3.	7 − 4	12 − 7	12 − 4	12 − 3	13 − 8	15 − 7	9 − 1	9 − 3	8 − 6	15 − 8
4.	1 − 1	10 − 6	3 − 1	13 − 6	12 − 8	4 − 2	11 − 7	8 − 0	10 − 3	6 − 6
5.	16 − 8	7 − 7	14 − 7	8 − 5	7 − 6	13 − 5	6 − 1	13 − 9	4 − 4	9 − 4
6.	8 − 1	3 − 0	8 − 4	9 − 9	4 − 1	15 − 6	10 − 9	5 − 0	3 − 3	14 − 8
7.	12 − 5	6 − 5	11 − 2	9 − 5	8 − 2	11 − 5	6 − 0	0 − 0	7 − 2	7 − 5
8.	9 − 6	10 − 2	16 − 9	10 − 7	6 − 2	14 − 9	8 − 3	11 − 4	11 − 8	5 − 2
9.	6 − 3	9 − 8	11 − 6	17 − 9	12 − 6	9 − 0	13 − 7	10 − 8	6 − 4	9 − 7
10.	1 − 0	7 − 3	5 − 4	11 − 9	5 − 3	15 − 9	10 − 4	3 − 2	18 − 9	5 − 1

Check your answers with those on page 51.

MULTIPLICATION

Terms You'll Need to Know

The numbers we multiply are called *factors*. When you multiply a *multiplicand* (the first number) by a *multiplier* (the second number), you get a *product* (the answer). You should be able to give the product of any two numbers between 1 and 12 instantly. If you can't, it's important to practice basic multiplication tables until your answers are immediate and correct (Figure 6).

When multiplying larger numbers, it's sometimes necessary to "regroup" or "carry" digits to the next place value.

Problem Solving

Example:

$$\begin{array}{r} 46 \\ \underline{\times\ 7} \end{array}$$

Multiply the 6 in the ones column by 7. Regroup or carry the 4.

$$\begin{array}{r} {}^{4}46 \\ \underline{\times\ 7} \\ 322 \end{array}$$

Multiply the 4 in the tens column by 7. Add the regrouped tens value of 4.

We also use zero as a "placeholder" when the multiplier isn't a single-digit number.

Example:

$$\begin{array}{r} {\scriptstyle 6\ 4}\ \ \\ {\scriptstyle \cancel{6}\ \cancel{5}}\ \ \\ 476 \\ \underline{\times\quad 89} \\ 4{,}284 \\ \underline{+\ 38{,}080} \\ 42{,}364 \end{array}$$

Multiply the multiplicand by 9.
Place a zero in the ones column before multiplying by 8 because the 8 is a tens value.
Add to find the product.

After a while, you'll be able to drop the zeros and just leave spaces. It's good to use zeros in the beginning so you don't lose any place values.

BASIC MULTIPLICATION TABLE

ZERO × ANY NUMBER = ZERO

1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40
1 × 11 = 11	2 × 11 = 22	3 × 11 = 33	4 × 11 = 44
1 × 12 = 12	2 × 12 = 24	3 × 12 = 36	4 × 12 = 48
5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72
5 × 10 = 50	6 × 10 = 60	7 × 10 = 70	8 × 10 = 80
5 × 11 = 55	6 × 11 = 66	7 × 11 = 77	8 × 11 = 88
5 × 12 = 60	6 × 12 = 72	7 × 12 = 84	8 × 12 = 96
9 × 1 = 9	10 × 1 = 10	11 × 1 = 11	12 × 1 = 12
9 × 2 = 18	10 × 2 = 20	11 × 2 = 22	12 × 2 = 24
9 × 3 = 27	10 × 3 = 30	11 × 3 = 33	12 × 3 = 36
9 × 4 = 36	10 × 4 = 40	11 × 4 = 44	12 × 4 = 48
9 × 5 = 45	10 × 5 = 50	11 × 5 = 55	12 × 5 = 60
9 × 6 = 54	10 × 6 = 60	11 × 6 = 66	12 × 6 = 72
9 × 7 = 63	10 × 7 = 70	11 × 7 = 77	12 × 7 = 84
9 × 8 = 72	10 × 8 = 80	11 × 8 = 88	12 × 8 = 96
9 × 9 = 81	10 × 9 = 90	11 × 9 = 99	12 × 9 = 108
9 × 10 = 90	10 × 10 = 100	11 × 10 = 110	12 × 10 = 120
9 × 11 = 99	10 × 11 = 110	11 × 11 = 121	12 × 11 = 132
9 × 12 = 108	10 × 12 = 120	11 × 12 = 132	12 × 12 = 144

FIGURE 6—Old-fashioned multiplication tables are still helpful when learning and reviewing the basics.

How to Check Multiplication

The best way to check multiplication is to multiply the numbers again. Use the number that was the multiplicand as the multiplier, and then let the number that was the multiplier be the multiplicand. In other words, if you multiplied 32×16, check your work by finding 16×32. If the results are the same, your work is correct. If your answers don't agree, check to find the error.

Multiplication can also be checked by division. (You'll be reviewing your division skills in the next section of this study unit.) To do this, you divide the product by either of the factors. For instance, if your multiplication problem is $16 \times 32 = 512$, you would check your solution this way: $512 \div 32 = 16$. You know your solution is correct, because 16 is the other factor of your multiplication problem.

It doesn't matter which method of checking you use. Just be sure to check your answers to *every* problem.

Before going on to refresh your skills in division, take time to complete *Count Your Change 4*.

Count Your Change 4

Write your answers to the exercises in the spaces provided. Practice until you can give all the answers correctly in five minutes or less.

1.	2×1	7×8	0×1	8×7	2×9	0×6	1×3	2×0	4×5	3×4
2.	4×3	6×7	7×1	8×8	9×2	5×5	4×6	7×0	4×0	7×4
3.	2×7	2×4	2×3	3×8	5×7	9×1	9×3	8×6	5×8	1×1
4.	0×6	3×1	3×6	2×8	4×2	1×7	8×0	0×3	6×6	6×8
5.	7×7	4×7	8×5	7×6	3×5	6×1	3×9	4×4	9×4	8×1
6.	3×0	8×4	9×9	4×1	5×6	0×9	5×0	3×3	4×8	2×5
7.	6×5	1×2	9×5	8×2	1×5	6×0	0×0	7×2	7×5	6×3
8.	0×2	6×9	0×7	6×2	4×9	8×3	1×4	1×8	5×2	6×3
9.	9×8	1×6	7×9	2×6	9×0	3×7	0×8	6×4	9×7	1×0
10.	7×3	5×4	1×9	5×3	5×9	0×4	3×2	8×9	5×1	2×2

Check your answers with those on page 51.

DIVISION

Terms You'll Need to Know

In division, you're finding the number of times one number is contained in another. In other words, you're separating a number into equal parts. For example, if you separate a foot-long (12-inch) hot dog into four equal parts, each part would be 3 inches long, since $12 \div 4 = 3$.

In the preceding equation, the *dividend* is the number to be divided (12); the *divisor* is the number by which you divide (4); and the *quotient* is the answer (3).

Short Division

Short division is a way of dividing small numbers in your head, as opposed to *long division*, which calls for keeping track of what you've done by writing it down step by step. In short division, only the divisor, dividend, and quotient are written. Instead of writing out columns of numbers, you'll carry out the operations mentally. Short division is sometimes easier to use than long division when the divisor is a small number, especially a single digit.

Let's solve the following problem using short division.

Problem Solving

Example: Divide 4,684 by 2.

Solution:

$$2\overline{)4684} \quad \text{quotient: } 2$$

Write the numbers as shown. Divide 2 into 4. Write 2 in the quotient.

$$2\overline{)4684} \quad \text{quotient: } 23$$

Divide 2 into 6. Write 3 in the quotient.

$$2\overline{)4684} \quad \text{quotient: } 234$$

Divide 2 into 8. Write 4 in the quotient.

$$2\overline{)4684} \quad \text{quotient: } 2342$$

Divide 2 into 4. Write 2 in the quotient.
Answer: 2,342

Long Division

Long division is a way of noting the *steps* and *answers* of longer problems. Long division is *not* more difficult than short division. It's just a way of jotting down everything you do so that you

- Don't lose track of the digits in the answer
- Can easily go back and check every step of your work

Problem Solving

Example: Let's say that you were throwing a banquet and had 416 guests coming. Your committee had collected \$73,216 to offset the expenses of the banquet. How much money do you have to spend on each guest?

Solution: Set up the problem as shown.

$$416\overline{)73216}$$

$$\begin{array}{r} 1 \\ 416\overline{)73216} \end{array}$$

You'll immediately notice that 416 certainly won't go into 7, nor will it go into 73. So, start by estimating how many times 416 will go into 732. If you quickly multiply 416 by 2, you'll realize that 416 won't even go into 732 twice. So, write 1 above the line, directly above the 2. (*Note:* Always write the quotient directly above the last digit of the number you divided into. In this case, you divided into 732, so the last digit is 2.)

$$\begin{array}{r} 1 \\ 416\overline{)73216} \\ \underline{-416} \end{array}$$

Multiply 416×1, and write 416 below 732 as shown.

$$\begin{array}{r} 17 \\ 416\overline{)73216} \\ \underline{-416} \\ 316 \end{array}$$

Subtract 416 from 732. (*Note:* If, at this time, the result of subtraction is greater than the divisor, you'll know you've estimated incorrectly, and need to increase your estimate. In this problem, 316 is less than the divisor 416, so you know your estimate is correct.)

$$\begin{array}{r} 17 \\ 416\overline{)73216} \\ \underline{-416\downarrow} \\ 3161 \end{array}$$

Now bring down the next digit (the 1) from the dividend. Estimate how many times 416 will divide into 3161. It might take you one or two tries to find that $416 \times 7 = 2912$. Write 7 above the line, directly above the 1.

$$\begin{array}{r} 17 \\ 416\overline{)73216} \\ \underline{-416} \\ 3161 \\ 2912 \end{array}$$

Multiply 416×7, and write 2912 under 3161.

$$\begin{array}{r} 17 \\ 416\overline{)73216} \\ \underline{-416} \\ 3161 \\ \underline{2912} \\ 2496 \\ \underline{2496} \\ 0 \end{array}$$

Subtract 2912 from 3161. Bring down the last digit (the 6). Estimate the number of times 416 will divide into 2496. You can figure that 416 goes into 2496 six times, so write 6 above the line directly above the 6 in the dividend. Multiply 416×6. Write 2496 as shown, and subtract. Since there are no more digits left in the dividend, you're finished with the division. *Answer:* There's $176 to spend on each guest attending the banquet.

So, you've determined that you have $176 to spend on each of those 416 guests. You'll be able to serve the finest wines, offer the finest foods, and hire the best entertainment. At any rate, having applied your knowledge of basic math, you're now assured that there won't be a lack of funds.

Example: Let's divide 42,126 by 42.

Solution: Set up the problem as shown.

$$42\overline{)42{,}126}$$

$$\begin{array}{r} 1 \\ 42\overline{)42126} \end{array}$$

First, divide the 42 into 42. You know that $42 \times 1 = 42$, so write 1 directly above the 2 as shown.

$$\begin{array}{r} 1 \\ 42\overline{)42126} \\ \underline{42} \\ 0 \end{array}$$

Multiply 42)1, and write 42 where shown. Subtract 42 from 42, and write 0 as shown.

$$\begin{array}{r} 10 \\ 42\overline{)42126} \\ \underline{42}\downarrow \\ 01 \end{array}$$

Bring down the next digit (the 1) from the dividend. Since 42 is too large to divide into 1 even once, write zero above the line, directly above the 1, as a placeholder. *Note:* This is a very important concept in division. Every time you bring down a digit from the dividend, you must write a number above the line, even if it's a zero.

$$\begin{array}{r} 100 \\ 42\overline{)42126} \\ \underline{42}\;\downarrow \\ 012 \end{array}$$

Bring down the next digit (the 2) from the dividend. Since 42 is too large to divide into 12 even once, write zero above the line, directly above the 2, as a placeholder.

$$\begin{array}{r} 1003 \\ 42\overline{)42126} \\ \underline{42} \\ 0126 \\ \underline{0126} \\ 0 \end{array}$$

Bring down the next digit (the 6) from the dividend. 42 will divide into 126 three times, so write a 3 directly above the 6. *Answer:* 42,126 divided by 42 equals 1003.

Remainders

If the divisor doesn't divide into the dividend exactly, the number left over is called the *remainder*. In quotients, remainders are labeled with the letter *r*. Let's take a look at a practice problem involving a quotient with remainder.

Problem Solving

Example: Divide 34,756 by 14.

Solution: Set up the problem as shown.

$$14\overline{)34756}$$

$$\begin{array}{rl}
 & 2 \\
14 & \overline{)34756} \\
 & \underline{28} \\
 & 6
\end{array}$$

Divide 14 into 34. When you multiply 14×2, you get 28. Write down 2 above the 4. Write 28 under 34 and subtract.

$$\begin{array}{rl}
 & 24 \\
14 & \overline{)34756} \\
 & \underline{28} \\
 & 67 \\
 & \underline{56}
\end{array}$$

Bring down the next digit (the 7) from the dividend and write it next to the 6 as shown. Divide 14 into 67. Since $14 \times 4 = 56$, you'll write 4 above the line in the hundreds place. Write 56 directly under 67 and subtract.

$$\begin{array}{rl}
 & 248 \\
14 & \overline{)34756} \\
 & \underline{28} \\
 & 67 \\
 & \underline{56} \\
 & 115 \\
 & \underline{112} \\
 & 3
\end{array}$$

Bring down the next digit (the 5) and write it next to the 11 as shown. Since $14 \times 8 = 112$, write 8 in the quotient and write 112 directly below the 115. Subtract.

$$\begin{array}{rl}
 & 2482 \; r\,8 \\
14 & \overline{)34756} \\
 & \underline{28} \\
 & 67 \\
 & \underline{56} \\
 & 115 \\
 & \underline{112} \\
 & 36 \\
 & \underline{28} \\
 & 8
\end{array}$$

Bring down the last digit (the 6) and write it next to the 3. Since $14 \times 2 = 28$, write a 2 in the quotient, and write 28 directly under the 36. Subtract. When you subtract, you'll have a difference of 8. This is the remainder of the division problem, since there are no more digits to bring down from the dividend.

Write the remainder of 8 in the quotient above the line as shown. *Answer:* 2482 *r* 8

How to Check Division

If your division is correct, the quotient multiplied by the divisor should equal the dividend. If we divide 704 by 32, we obtain 22. To check our answer, we multiply 32)22 and get 704. This proves that our division problem is correct.

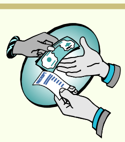
If the division had not been exact-that is, if there had been a remainder-we would have added the remainder to the product in our checking procedure. For example,

```
      2482 r 18
94 )12708
     94
      330
      282
       488
       470
         8  remainder
```

To check: $94 \times 135 = 12690$; $12690 + 18 = 12708$

Of course, if your check doesn't verify your answer, you must go back over your work to find the error.

Before going on to refresh your skills using decimals, take time to complete *Count Your Change 5*.



Count Your Change 5

Write your answers to the exercises in the spaces provided.

1. 61,982 23 ________
2. 803 37 ________
3. 816 28 ________
4. 219 47 ________
5. 2,624 82 ________
6. 50,000 72 ________
7. 1,333 43 ________
8. 6,528 16 ________

Check your answers with those on page 51.

DECIMALS AND FRACTIONS

A Look at Decimals

A *decimal number* is a fraction of a whole number that has a power of 10 as its denominator. Because it's in base 10, it doesn't need to be expressed as a fraction value. A *decimal point* is the dot (period) written between two digits to separate the fractional part of a number from the rest of that number. For example, $25.99 represents the fraction $25\frac{99}{100}$.

Performing operations with decimals is very similar to working with whole numbers. You simply have to remember the rules for the decimal point in each operation. In addition or subtraction, you have to be sure to "line up" the decimal points before performing the desired operation. Then bring the decimal point into the answer.

You can include zeros to fill in place values after the decimals. This makes it easier to keep the numbers in line.

Example: 4.87 + 17.667 + 20 + 6.5

$$\begin{array}{r} 4.870 \\ 17.667 \\ 20.000 \\ +\ \ 6.500 \\ \hline 49.037 \end{array}$$

Example: 16.8 – 2.437

$$\begin{array}{r} 16.800 \\ -\ \ 2.437 \\ \hline 14.363 \end{array}$$

When multiplying by decimals, you perform the operation as you did with whole numbers. Then you count the decimal places in the two factors. This will be the number of decimal places in the product.

Example:

$$\begin{array}{rl} 2.46 & \text{2 decimal places} \\ \times\ 1.3 & \text{1 decimal place} \\ \hline 738 & \\ +\ 2460 & \\ 3.198 & \text{3 decimal places in the product} \end{array}$$

Dividing decimals can be tricky when your divisor is a decimal. Otherwise, you simply divide as you did with whole numbers and bring your decimal up into your quotient.

Your decimal point simply "moves up" into the quotient.

Example: 49.2 ÷ 12

$$\begin{array}{r} 4.1 \\ 12\overline{)49.2} \\ \underline{48} \\ 1\,2 \\ \underline{1\,2} \\ 0 \end{array}$$

When your divisor is a decimal, you have to "remove" the decimal point by moving it to the right. You then have to move the decimal point in the dividend the same number of places.

Move the decimal in your divisor one place to make it a whole number. Then, move the decimal in your dividend one place also. Divide as before, bringing the decimal point up into the quotient.

Example: 37.92 ÷ 1.2

$$\begin{array}{r} 31.6 \\ 1.2.\overline{)37.9.2} \\ \underline{36} \\ 19 \\ \underline{12} \\ 72 \\ \underline{72} \\ 0 \end{array}$$

A Look at Fractions

A fraction is a numerical way for naming a part of a whole. There are two terms in a fraction. The top of the fraction is the *numerator*. The bottom is called the *denominator*.

There are three types of fractions you should know. The first is a *proper fraction,* which is a fraction in which the numerator is less than the denominator. The second is an *improper fraction* where the numerator is greater than or equal to the denominator. Finally, there are *mixed numbers.* Mixed numbers include both a whole number and a fraction.

Example:

$\frac{2}{3}, \frac{3}{5}, \frac{7}{9}, \frac{1}{10}$	proper fractions
$\frac{5}{2}, \frac{8}{3}, \frac{10}{7}, \frac{4}{4}$	improper fractions
$2\frac{1}{3}, 3\frac{1}{4}, 10\frac{7}{8}, 1\frac{5}{6}$	mixed numbers

You can change an improper fraction to a mixed number by dividing the denominator into the numerator. The remainder is placed as a numerator over the original denominator.

Problem Solving

Example: Change the improper fraction $\frac{8}{3}$ to a mixed number.

Solution: Set up the fraction $\frac{8}{3}$ as a division problem.

$\frac{8}{3} = 8 \div 3$	The fraction $\frac{8}{3}$ can be read "8 divided by 3."
$8 \div 3 = 2\ r\ 2$	Divide the denominator into the numerator.
$2\frac{2}{3}$	The quotient 2 is the whole number part of your answer. The remainder, 2, is the numerator of the fraction part of your answer. The divisor 3 is the denominator of the fraction part. *Answer:* $2\frac{2}{3}$

A mixed number can also be changed into an improper fraction by multiplying the denominator by the whole number. You then add the product to the numerator, and place the total over the denominator.

Problem Solving

Example: Change $3\frac{1}{4}$ to an improper fraction.

Solution: Set up the problem as shown.

$4 \times 3 = 12$	Multiply the denominator 4 by the whole number 3.
$1 + 12 = 13$	Add the numerator to the product.
$3\frac{1}{4} = \frac{13}{4}$	Place this number over the denominator

Fractions that are equal to each other are knownas equivalent fractions. You can find an equivalent fraction by multiplying the numerator and denominator of a fraction by the same number.

Problem Solving

Example:

$$\frac{2}{3}=\frac{2\times 2}{3\times 2}=\frac{4}{6}$$

$\frac{2}{3}$ and $\frac{4}{6}$ are equivalent.

$$\frac{3}{5}=\frac{3\times 6}{5\times 6}=\frac{18}{30}$$

$\frac{3}{5}$ and $\frac{18}{30}$ are equivalent.

Adding and Subtracting Fractions

When adding and subtracting fractions, the fractions must have the same denominator. If the denominators are equal, you simply add or subtract the numerators and bring over the denominator.

Example:

$$\frac{2}{7}+\frac{3}{7}=\frac{5}{7}$$

Add or subtract the numerators and put the result over the denominator.

$$\frac{5}{9}-\frac{4}{9}=\frac{1}{9}$$

If the fractions are "unlike," that is, they have different denominators, then you must make equivalent fractions with the same denominators. First, you need to find the least common denominator (LCD) for each fraction. Then you express the fractions in terms of this LCD.

The easiest way to find the LCD is to look at multiples of the largest denominator. Then find the first one that the smaller denominators divide into evenly.

Example:

$$\frac{1}{2}+\frac{3}{4}+\frac{5}{8}+\frac{5}{6}$$

The largest denominator is 8.

8, 16, 24, 32, 40 . . . List the multiples of 8.

LCD = 24 The smallest multiple of 8 that 2, 4, and 6 will divide into evenly is 24. This is the least common denominator (LCD).

Now change each fraction into an equivalent fraction with 24 as the denominator.

Example:

$$\frac{1}{2}=\frac{1\times 12}{2\times 12}=\frac{12}{24}$$

To get a denominator of 24, multiply both numerator and denominator of $\frac{1}{2}$ by 12 ($2\times 12=24$).

$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$ To get a denominator of 24, multiply both numerator and denominator of $\frac{3}{4}$ by 6 ($4 \times 6 = 24$).

$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$ To get a denominator of 24, multiply both numerator and denominator of $\frac{5}{8}$ by 3 ($8 \times 3 = 24$).

$\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$ To get a denominator of 24, multiply both numerator and denominator of $\frac{5}{6}$ by 4 ($6 \times 4 = 24$).

Finally, add your new fractions together as before. Express your final answer as a mixed number.

$\frac{12}{24} + \frac{18}{24} + \frac{15}{24} + \frac{20}{24} = \frac{65}{24} = 2\frac{17}{24}$ Add the numerators and place the sum above the denominator of 24. Change the fraction to a mixed number. *Answer:* $2\frac{17}{24}$

You often need to find the LCD when adding or subtracting mixed numbers. First you need to find the LCD so that you can add or subtract the fractions. Then add or subtract the whole numbers. Combine these two parts together for the final answer.

Multiplying Fractions

Multiplying fractions is easier since you don't need to have like denominators. You simply multiply the numerators together first. Then put this number over the product of the denominators.

Example:

$\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$ Multiply the numerators ($1 \times 3 = 3$). Multiply the denominators ($2 \times 4 = 8$).

$\frac{2}{5} \times \frac{2}{3} = \frac{2 \times 2}{5 \times 3} = \frac{4}{15}$ Multiply the numerators ($2 \times 2 = 4$).Multiply the denominators ($5 \times 3 = 15$).

Dividing Fractions

Dividing fractions is similar to multiplying fractions. However, there's one step you need to remember before multiplying. You first have to *invert* (flip upside down) the multiplier (the second fraction). This is called the *reciprocal*. After you find the reciprocal, you multiply as before.

Example:

$\frac{1}{2} \div \frac{3}{5} = \frac{1}{2} \times \frac{5}{3}$ Invert the multiplier (the second fraction), and set up as a multiplication problem as shown.

$\frac{1}{2} \times \frac{5}{3} = \frac{5}{6}$ Multiply.

Sometimes when multiplying or dividing fractions, you can *cross-cancel before performing the desired operation. That is, if a numerator divides evenly into a denominator, you can cancel* the numbers by dividing the smaller one into the larger one. *Cancellation* is a way of reducing terms for easier multiplication or division.

Example:

$\frac{3}{5} \times \frac{15}{24}$ Using the cancellation method, you'll see that the 3 in the numerator of the first fraction divides evenly into the denominator of the second fraction (24).

$\frac{\overset{1}{\cancel{3}}}{5} \times \frac{15}{\underset{8}{\cancel{24}}}$ Draw a line through the 3 and write a 1 above it, and draw a line through the 24 and write an 8 below it as shown. You'll also see that the 5 in the denominator of the first fraction divides evenly into the numerator of the second fraction (15).

$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{8}{\cancel{24}}}$ Draw a line through the 5 and write a 1 below it, and draw a line through the 15 and write a 3 above it as shown. Now, multiply the numerators $(1 \times 3 = 3)$. Also, multiply the denominators $(1 \times 8 = 8)$. *Answer:* $\frac{3}{8}$.

Also, if you're multiplying or dividing mixed numbers, you must first change them into improper fractions.

Example:

$2\frac{1}{2} \div 3\frac{1}{4}$ Write out the problem.

$\frac{5}{2} \quad \frac{13}{4}$ Change mixed numbers to improper fractions

$\frac{5}{2} \times \frac{4}{13}$ Multiply $\frac{5}{2}$ by the reciprocal of $\frac{13}{4}$ $\left(\frac{4}{13}\right)$.

$\frac{5}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{13}$ Cross-cancel and multiply to get $\frac{10}{13}$.

Before going on to refresh your skills at rounding off results, take time to complete *Count Your Change 6*.

Count Your Change 6

A. Add the following problems involving decimals.

1. 1.4
   1.8

2. 3.40
   5.39

3. 6.95
   8.64
   5.30

4. 3.643
   8.921

5. 7.62
   8.95

6. 1.89
   8.22

7. 26.70
   41.56

B. Subtract the following problems involving decimals.

1. 725.65
   – 618.25

2. 1060.22
   – 656.70

3. 475.91
   – 238.12

C. Complete the following multiplication problems involving decimals.

1. 3.278×0.145
2. 5037×0.0196
3. 784.3×16.88
4. 3968×0.1035
5. 82.49×133.9

D. Complete the following division problems involving decimals.

1. $0.03125 \div 125$
2. $525 \div 0.035$
3. $16.80 \div 96$
4. $0.7864 \div 80$

(Continued)

Count Your Change 6

E. Add the fractions below. Remember to find the LCD first.

1. $\frac{1}{2} + \frac{1}{3}$

2. $\frac{1}{6} + \frac{1}{8} + \frac{1}{3}$

3. $\frac{4}{11} + \frac{5}{22} + \frac{3}{44}$

4. $\frac{1}{5} + \frac{1}{7} + \frac{3}{21}$

5. $\frac{1}{4} + \frac{2}{5} + \frac{1}{3}$

F. Subtract the fractions below. Remember to find the LCD first.

1. $\frac{3}{4} - \frac{3}{8}$

2. $\frac{11}{15} - \frac{3}{5}$

3. $\frac{3}{4} - \frac{1}{6}$

4. $\frac{10}{17} - \frac{6}{17}$

G. Multiply the following fractions.

1. $\frac{1}{2} \times \frac{2}{3}$

2. $1\frac{1}{2} \times \frac{3}{4}$

3. $\frac{4}{4} \times \frac{7}{6}$

4. $\frac{12}{25} \times \frac{5}{6} \times \frac{3}{4}$

5. $\frac{13}{7} \times \frac{49}{2} \times \frac{4}{39}$

6. $\frac{11}{15} \times \frac{3}{5} \times \frac{25}{7}$

H. Divide the following fractions. Remember to invert the multiplier, then multiply the fractions.

1. $3 \div \frac{1}{2}$

2. $\frac{2}{3} \div \frac{1}{2}$

3. $\frac{10}{7} \div \frac{4}{7}$

4. $\frac{3}{4} \div \frac{5}{7}$

Check your answers with those on page 52.

ROUNDING OFF RESULTS

You've already discovered how useful rounding off is when performing long division. Once you learn how to round off correctly, you'll probably be surprised at how often it can be applied to everyday situations.

At times, you'll find numbers that must be expressed in certain units. A common example is dollars and cents. A storekeeper who sells two cans of peas for \$0.53 can't accept 26 $\frac{1}{2}$ cents for one can. As you well know, he will charge you \$0.27. Or if two pounds of coffee cost \$4.55, you'll have to pay \$2.28 for one pound even though $\frac{1}{2} \times \$4.55$ is \$2.27 $\frac{1}{2}$. The storekeeper must round off the price to the nearest cent, since a cent is the smallest unit of our money system.

Rounding off is expressing a number in the nearest *whole* number of units, such as cents in the preceding examples. When you round off a number, you drop, or discard, all the digits beyond a certain point. If the first digit *of the part you drop* is a 5 or greater than 5, you add 1 to the last digit retained. If the first digit of the dropped part is smaller than 5, you leave the last retained digit as it is.

Suppose that you're to round off 367,423 to the nearest *thousand*. Here you drop the figures 423, and since 4 is less than 5, you won't change the last digit of the part to be retained. Thus, 367,423 rounded off to thousands becomes 367,000. Notice that zeros are put in the place of the three discarded digits. If this had not been done, the *magnitude* (size) of the original number would have been changed; it would have been 367 instead of 367,000. You must be very careful not to change the magnitude of a number by merely dropping digits. Guard against the common error of saying, for example, that 367,423 rounds off to 367.

On the other hand, if you drop figures to the *right* of the decimal point, it isn't necessary to replace them by zeros. The reason for this is that the magnitude of the original number isn't changed. For example, if you round off 325.473 to the nearest *tenth*, it becomes 325.5. Note that dropping the last two decimal places doesn't change the magnitude of the original number.

Now let's consider rounding off 426,842 to thousands. Here, since 8 is greater than 5, you increase the 6 to 7, and the rounded-off number becomes 427,000. Again, zeros are used to retain the magnitude of the original number.

Example: Round to the nearest thousands.

537923	7 is in the thousands place. Look at the number to the right of 7. Since 9 is greater than 5, you increase 7 to 8.
538,000	Replace everything after 8 with a zero.

MENTAL MATH AT THE CHECKOUT

Have you ever had the experience of tossing items into your shopping cart and wondering, as you approached the checkout counter, whether you had enough cash to pay for those items (Figure 7)? You can obtain a rough estimate of their cost with some quick mental math.

FIGURE 7—Having a rough idea of the total of your purchases can save you the embarrassment of being "caught short" at the checkout.

Since most prices are odd amounts ($0.67, $3.99, and so on), they should be rounded off to make your mental math easier. The cents can be rounded off to the nearest dime, the nearest quarter, or the nearest half-dollar. Rounding off to the nearest half-dollar is easiest for most people, but use whichever method is easiest for *you*. Mentally add the first two rounded-off prices, add the third one to that total, and so on until they're all added.

(Continued)

MENTAL MATH AT THE CHECKOUT *(Continued)*

Here's an example. Your cart contains items with the following prices: \$3.84, \$1.67, \$3.90, \$0.38, \$0.76, and \$6.20. Depending on which unit of money you use as a basis for rounding off, your mental math gives you these results:

Rounded off to the nearest dime	Rounded off to the nearest quarter	Rounded off to the nearest half-dollar
\$3.84 = \$ 3.80	\$3.84 = \$4.00	\$3.84 = \$ 4.00
+ 1.67 = 1.70	+ 1.67 = 1.75	+ 1.67 = 1.50
5.50	5.75	5.50
+ 3.90 = 3.90	+ 3.90 = 4.00	+ 3.90 = 4.00
9.40	9.75	9.50
+ 0.38 = 0.40	+ 0.38 = 0.50	+ 0.38 = 0.50
9.80	10.25	10.00
+ 0.76 = .80	+ 0.76 = .75	+ 0.76 = 1.00
10.60	11.00	11.00
+ 6.20 = 6.20	+ 6.20 = 6.25	+ 6.20 = 6.00
\$16.80	\$17.25	\$17.00

The actual cost of the items is \$16.75. As you can see, all of the rough estimates (\$16.80, \$17.25, and \$17.00) came close to the actual total.

If the items you purchase are taxed, round off your mental total to the nearest dollar. Then multiply that figure by the percent of tax. For example, if you obtained a total of \$16.80, round it off to \$17.00. If the tax is 6%, mentally multiply 17 × 6 = 102; move the decimal to the left two places to obtain \$1.02, or \$1.00, a rough estimate of the tax. Now you know you'll need approximately \$17.00 + \$1.00, or \$18.00, to pay the cashier.

This mental total will also help you to run a quick check on what you've been charged. If the amount charged seems much larger than you estimated, you can ask for a verification of your receipt. Or you can check later to be sure you weren't overcharged.

It's an excellent idea to practice this mental math a few times at home, using old cash register receipts containing several purchases. Round off the price of each item just as you would if the items were in your cart, adding them as you go along. Your mental total should be within a dollar or two of the actual total shown on the receipt.

You can use such mental math in many situations—when you want a rough estimate of the cost of items, the quantity of materials you'll need, the length of time a certain job will take, and so on. Use this skill whenever you have the opportunity. Soon, you'll be using mental math automatically in everyday situations, saving yourself time, effort, and, quite possibly, money.

Count Your Change 7

A. Round off the following numbers to the nearest thousand.

1. 345,276 _______
2. 127,825 _______
3. 674,620 _______
4. 355,500 _______

B. Round off the following numbers to the nearest tenth.

1. 42.51 _______
2. 39.62 _______
3. 26.425 _______
4. 47.546 _______

C. Round off the following numbers to the nearest cent and then to the nearest dollar.

1. $365.473 _______
2. $125.654 _______
3. $62.756 _______
4. $38.415 _______

Check your answers with those on page 53.

SOLVING PERCENT PROBLEMS

Terms and Symbols You'll Need to Know

Everyone enjoys hearing, "You'll get a five-percent increase in your salary." Most raises are calculated as a percentage of your present wages (Figure 8).

The term *percent* is used so frequently in everyday life that you are, no doubt, already familiar with its meaning. You must also be able to handle calculations in which percent is involved. Such calculations are needed in expressing profits, losses, depreciation, discounts, and many other money operations. The word percent is a shortened form of the Latin words *per centum*, which mean "by the hundred."

The symbol for percent is %. Thus, 2% is read "2 percent," and it means $\frac{2}{100}$ or 0.02. Notice, from the preceding sentence, that the decimal point is merely moved two places to the left to write a percent as a decimal. In most calculations with a percent, you must express it as a fraction or as a decimal.

FIGURE 8—Most raises are calculated as a percentage of your present wages.

How to Express Percent

If you remember that 12% means $\frac{12}{100}$ or 0.12, you should have no trouble changing a percent to either a fraction or a decimal. First, you drop the percent sign. Next, to change a percent to a fraction, you write the number indicating percent as the numerator of a fraction having a denominator of 100. Then, reduce the fraction to its lowest terms ($25\% = \frac{25}{100} = \frac{1}{4}$).

Changing Percents to Decimals

You already know that decimals with two decimal places represent hundreths. The decimal 0.23, for example, represents 23 hundreths ($\frac{23}{100}$). Percents are another way of expressing hundreths. So, since $23\% = \frac{23}{100}$, we can also say that 23% = 0.23.

To change any percent to a decimal or mixed decimal, drop the % sign and move the decimal point two places *to the left.* Add zeroes as placeholders if necessary.

Example:

45%	Write the percent.
45	Drop the percent sign.
45.	Move the decimal point.
.45	45% = .45 as a decimal.

To change a percent that includes a fraction, such as $3\frac{1}{2}\%$, into a decimal, first drop the percent sign. Then, change the mixed number to a decimal. (Remember: to change a fraction to a decimal, divide the numerator by the denominator.)

Example:

$2\frac{1}{2}\%$	Write the percent.
$2\frac{1}{2}$	Drop the percent sign.
$2\frac{1}{2} = 2.5$	Change the mixed number to a decimal.
02.5	Move the decimal point.
.025	$2\frac{1}{2} = .025$ as a decimal.

Changing Decimals to Percents

You've just learned how to change a percent to a decimal. Now, you'll learn how to do the reverse process. To change any decimal or mixed decimal to a percent, move the decimal point two places to the *right*, adding zeroes as place holders if necessary. Then, add a % sign.

Note: These steps are just the reverse of the steps for changing a percent to a decimal.

.85	Write the decimal.
.85	Move the decimal point.
85%	Add the % sign.

Problem Solving

Example: The suggested tipping rate in a certain restaurant is .15. What is this rate as a percent?

.15	Move the decimal point two places to the right.
15%	Add the percent symbol. *Answer:* The suggested rate for tips is 15%.

Changing Percents to Fractions

To change any percent to a fraction or a mixed number drop the percent sign. The number will be the *numerator* of the fraction. The *denominator* is 100. Reduce the fraction to its simplest form, if necessary.

Problem Solving

Example: Change 48% to a fraction.

48	Drop the percent symbol.
$\frac{48}{100}$	The number 48 is the numerator of the fraction, and 100 is the denominator.
$\frac{48\ \ 4}{100\ \ 4} = \frac{12}{25}$	Reduce the fraction. *Answer:* 48% equals the fraction $\frac{12}{25}$.

Changing Fractions to Percents

If the denominator of your fraction is a factor of 100, your task is very simple. (A *factor* is a number that goes evenly into another number, with no remainder.) You multiply the denominator and the numerator by the number of times the denominator goes into 100.

For instance, suppose you have to express $\frac{3}{25}$ as a percent. 25 goes into 100 four times exactly, with no remainder. So, you multiply

$$\frac{3}{25} \times \frac{4}{4} = \frac{12}{100} = 12\%$$

If your denominator is *not* a factor of 100, it won't go evenly into 100, so you need to use a different method. Your first step is to convert the fraction to a decimal. To do this, simply divide the numerator by the denominator.

For example, say you want to express $\frac{5}{8}$ as a percent. Your first step is to divide 5 by 8. Since your division skills are up to par, this should be no problem. Your work should look like this:

$$\begin{array}{r} .625 \\ 8\overline{)5.0} \\ \underline{4.8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Often it's easier to use fractions when doing computations involving percents. See Figure 9 for conversions of some of the most common percentages.

FIGURE 9—The fractional equivalents of some often used percents are shown here

$50\% = \frac{1}{2}$	$40\% = \frac{2}{5}$	$37\frac{1}{2}\% = \frac{3}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$
$25\% = \frac{1}{4}$	$60\% = \frac{3}{5}$	$62\frac{1}{2}\% = \frac{5}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$
$75\% = \frac{3}{4}$	$80\% = \frac{4}{5}$	$87\frac{1}{2}\% = \frac{7}{8}$	$83\frac{1}{3}\% = \frac{5}{6}$
$20\% = \frac{1}{5}$	$12\frac{1}{2}\% = \frac{1}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$	

You've now determined that the fraction $\frac{5}{8}$ equals the decimal 0.625. From this point, it's an easy step to convert 0.625 to a percent. To express *any* decimal as a percent, just move the decimal point two places to the right and add the percent sign. Thus, $0.625 = 62.5\%$. Since you've determined that $\frac{5}{8} = 0.625$ and $0.625 = 62.5\%$, you now know that $\frac{5}{8} = 62.5\%$.

Finding Percentages of Any Number

In all your work with percent, the basic relation $p = br$ will be the key. In this formula, p stands for *percentage*, b for the *base*, and r for the *rate*, or percent. It's important not to confuse *percentage* with *percent*. Percentage is an *amount*-the product of a percent and a number that's called the base.

By solving the formula $p = br$, you can very easily find the percentage, base, or percent of any problem. Let's take a look at an example of each.

First, let's find the *percentage* when the percent and base are given, using the formula $p = br$.

Example: Find 25% of \$250.

Solution: Use the formula: $p = br$ where

p = unknown (percentage)
b = \$250 (base)
r = 25% (percent or rate)

$p = br$ $p = \$250 \times 25\%$	Write the formula. Substitute the known values into the formula.
$250 \times 0.25 = \$62.50$	Perform the calculation. *Answer:* The percentage is \$62.50.

Now let's find the *percent* when the percentage and base are given, using the formula $r = \frac{p}{b}$.

Example: What percent of \$54 is \$18?

Solution: Use the formula $r = \frac{p}{b}$ where

r = unknown (percent or rate)
p = \$18 (percentage)
b = \$54 (base)

$r = \frac{p}{b}$ — Write the formula.

$r = \frac{18}{54}$ — Substitute the known values into the formula.

$\frac{18}{54} = \frac{1}{3} = 0.333 = 33\frac{1}{3}\%$ — Perform the calculations. *Answer:* The percent is $33\frac{1}{3}\%$.

Finally, let's find the *base* when percentage and percent are given, using the formula $b = \frac{p}{r}$.

Example: In a survey, 30% of the people surveyed were retired. If there were 90 retirees in the survey, how many people were surveyed?

Solution: Use the formula $b = \frac{p}{r}$ where

b = unknown (base)
p = 90 (percentage)
r = 30% (percent or rate)

$b = \frac{p}{r}$ — Write the formula.

$b = \frac{90}{30\%}$ — Substitute the known values into the formula.

$\frac{90}{30\%} = \frac{90}{0.30} = 300$ — Change the percent to a decimal. Perform the calculations.

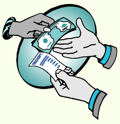
$b = 300$ — *Answer:* 300 people were surveyed.

Terms Used in Problems of Percent

In using percents, you'll discover many expressions that you'll have to interpret very carefully. Yet all of these are quite easy to understand if you use common sense in reading them. For example, 20% more than 40 means 40 + (20% of 40) = 40 + 8 = 48. Similarly, 10% less than 40 means 40 – (10% of 40) = 36.

If you were asked to find the percent increase or decrease in two numbers, you would first find the actual increase or decrease, and then you would find what percent it is of the number you started with. For example, if your utility bills for November totaled $250, and for December totaled $300, there was an increase of $50. This is an increase of $\frac{50}{250} = 0.20 = 20\%$. Likewise, if your utility bills for January were \$270, this would mean a decrease of \$30 from those of December. This is a decrease of $\frac{30}{300} = 0.10 = 10\%$.

Before going on to learn about discounts, take time to complete *Count Your Change 8*.



Count Your Change 8

A. Write each of the following as a fraction and then as a decimal.

		Fraction	Decimal			Fraction	Decimal
1.	25%	______	______	7.	12.5%	______	______
2.	32.5%	______	______	8.	0.2%	______	______
3.	4%	______	______	9.	0.75%	______	______
4.	75%	______	______	10.	107%	______	______
5.	6.5%	______	______	11.	210%	______	______
6.	125%	______	______	12.	22.5%	______	______

B. Express each of the following as a percent.

		Percent			Percent
1.	$\frac{4}{10}$	______	7.	$\frac{31}{100}$	______
2.	$3\frac{1}{4}$	______	8.	$1\frac{3}{4}$	______
3.	$\frac{3}{10}$	______	9.	$22\frac{1}{3}$	______
4.	$\frac{3}{8}$	______	10.	$2\frac{7}{8}$	______
5.	$7\frac{26}{100}$	______	11.	$\frac{7}{8}$	______
6.	$\frac{1}{8}$	______	12.	$\frac{1}{800}$	______

C. Find the percentage in the following. Write your answers on a separate sheet of paper, labeling the base, rate, and percentage in your work.

1. 25% of 800
2. 45% of 70
3. $33\frac{1}{3}\%$ of 120
4. $5\frac{1}{2}\%$ of \$600
5. 125% of 580 bu (bushels)
6. $2\frac{1}{2}\%$ of 740 cases
7. $\frac{1}{4}\%$ of 2764 ft (feet)
8. 4.25% of 6784 lb (pounds)
9. 20% of 972 in. (inches)
10. $37\frac{1}{2}\%$ of 96 min (minutes)
11. $66\frac{2}{3}\%$ of 720 tons
12. $3\frac{1}{2}\%$ of 450

(Continued)

Count Your Change 8

D. Find the rate, or percent, in the following, accurate to the nearest 0.1%.

1. Base 72; percentage 12 ________
2. Percentage 28; base 224 ________
3. Base 20; percentage 40 ________
4. Base 44; percentage 99 ________
5. Percentage 126; base 8400 ________

E. Find the base in the following.

1. 50 is 20% of what number? ________
2. 18 is $33\frac{1}{3}\%$ of what number? ________
3. 4 is 1% of what number? ________
4. 5% of what number is 90? ________
5. $16\frac{2}{3}\%$ of what number is 72? ________

F. Express your answers to the following questions to the nearest 0.1%.

1. 20 is how much less than 30? What percent less? ________________
2. 25 is how much more than 20? What percent more? ________________
3. 37 is what percent more than 32? ________________
4. 300 is what percent less than 599? ________________
5. 184 is what percent more than 148? ________________
6. 102 is what percent more than 50? ________________
7. 16 is what percent less than 489? ________________
8. 216 is what percent greater than 80? ________________
9. 418 is what percent smaller than 516? ________________
10. A class had 30 pupils at the beginning of this school term, but now has 5 more pupils. What is the percent of increase? ________________
11. Eggs were 65¢ per dozen on January 1 and 60¢ per dozen on February 1. What percent did the price decrease during January? ________________
12. During November the price of each share of a certain stock declined from $196 to $132. What was the rate of decline for that month? ________________
13. By careful reorganization, a company was able to reduce the cost of operation from $21,840 per month to $17,190 per month. By what percent was the cost of operation reduced? ________________
14. The circulation of a city library for September was 157,928 books; for October, 184,211 books. What was the percent of gain for October over September? ________________

Check your answers with those on page 53.

DISCOUNTS

Terms You'll Need to Know

You've seen many, many advertisements of special sales. Usually, the sale offers certain articles at a reduced price (Figure 10). This reduced price can be stated in two ways, for example,

Men's jackets reduced 25%

or

Men's $50 jackets reduced to $37.50

FIGURE 10—Sales advertisements are everywhere. But how much will you really save?

The reduction in price is called a *discount* or a *markdown*. The original price at which goods are sold, without any reduction, is the *marked price*. The price at which the goods are sold is the *selling price*. When there's no discount, the selling price and the marked price are the same.

Retail discounts like those just mentioned are often expressed in percent. When discounts are given in percent, they're always applied to the marked price. For example, the selling price of a chair marked $75 subject to a discount of 20% is found as follows:

Marked price	= $ 75
Discount (20% of $75)	= – 15
Selling price	= $ 60

If you don't need to know the amount of money involved in the discount, the selling price can be obtained in a simpler manner. Subtract the percent of discount from 100%, and then multiply the remainder by the marked price.

Using the previous example, the marked price is $75 and the discount is 20%.

$$100\% = 1.00$$
$$20\% = 0.20$$
$$1.00 - 0.20 = 0.80$$
$$\$75 \times 0.80 = \$60.00 \text{ selling price}$$

This is the same answer you obtained by the first method. Use this easier method whenever possible.

Trade Discounts

To become a knowledgeable consumer-getting the most for your money-it's important to understand pricing. How is the final selling price of that new car, suit, or stereo determined? And who determines it?

Manufacturers and wholesalers (businesses that sell goods to other businesses) give discounts that are sometimes called *trade discounts*. Trade discounts are reductions on the prices shown in the manufacturers' or wholesalers' catalogs. These discounts are given to retail businesspeople who sell merchandise directly to you, the consumer.

To make frequent printings of the catalogs unnecessary, the list prices shown in them are much above the normal selling prices. Then, by issuing *discount sheets*, the wholesaler can adjust the listed prices to the prevailing market. For example, if the list price for a certain item is double its selling price, the discount will be 50%. Later on, if the selling price of this item changes, another discount notice will quote a new discount. Thus, if the selling price increases, the discount will be decreased to, let's say, 42%. Or if the selling price decreases, the discount will be increased to something like 55%. By using discount sheets, the wholesaler eliminates the expense of making out a new price list every time prices change.

Discount Series

Often, several discounts are quoted by a wholesaler. When there's more than one discount, the discounts are called a *discount series*. The first discount is a percent of the list price. The second discount is a percent of the remainder after the first discount has been subtracted from the list price, and so on. To find the selling price, multiply the list price by the first percent and subtract the discount from the list price. Compute the second discount, using the first remainder as a base, and subtract the second document from the first remainder. Repeat this process, using each remainder as the base for computing the next discount. The last remainder is the selling price (Figure 11).

SCOTT
PLUMBING SUPPLIES
754 Walnut Street, Philadelphia PA 19106

February 15, 19—

Sold to: F.R. Brooks & Son
25 Monroe Ave
Newburgh, NY 10504

8	Faucet Units, No. LM 745 @$92.46, less 25%, 10%	489.29
12	Faucet Units, No. BM 627 @ $51.27, less 30%, 5%	409.13
	Total	**898. 42**

FIGURE 11—This bill shows the total discounted selling price.

Let's look at a couple of example problems to illustrate this procedure.

Example:

Suppose you wanted to find the selling price of an item listed at $300 which is subject to discounts of 40%, 10%, and 5%. What process would you follow?

Solution:

List Price	$300.00
First discount (40% of $300)	120.00
Remainder after first discount	180.00
Second discount (10% of $180)	18.00
Remainder after second discount	162.00
Third discount (5% of $162)	8.10
Selling Price (answer)	$153.90

Example:

You want to determine the selling price of a bill of goods amounting to $720, on which discounts of 30%, 10%, and 5% are allowed.

Solution:

List price	\$720.00
First discount (30% of \$720)	216.00
Remainder after first discount	504.00
Second discount (10% of \$504)	50.40
Remainder after second discount	453.60
Third discount (5% of \$453.60)	22.68
Selling price (answer)	\$430.92

Reducing a Series to a Single Discount

When a discount series is allowed, most businesspeople and consumers reduce it to a single discount so that they can get the final selling price in one operation. If there are a great number of bills to be figured, you can see how this shorter method will save a lot of labor.

Fortunately, it's a very simple matter to express a discount series as a single discount. This is what you do: Subtract each rate of discount from 100%. Then multiply the remainders together. Subtract this product from 100%. The remainder will be the single discount.

Let's recalculate our last two example problems using this method.

In our first example, we wanted to find the selling price of an item listed at \$300, which is subject to discounts of 40%, 10%, and 5%. To find the selling price by reducing the discount series to a single discount, we first subtract each of the discount rates from 100%, leaving us with 60%, 90%, and 95%, respectively. We then convert these percentages to decimal form and follow this procedure:

$$0.60 \times 0.90 \times 0.95 = 0.513$$
$$100\% = 1.00$$
$$1.00 - 0.513 = 0.487$$
$$0.487 \times 300 = 146.10$$
$$300 - 146.10 = 153.90$$

Thus, the discount allowed is \$146.10, and the selling price is \$153.90, the same amount we calculated using the first method.

In our second example, we wanted to find the selling price of a bill of goods amounting to \$720, on which discounts of 30%, 10%, and 5% were allowed. By subtracting each of the given discount rates from 100%, we're left with 70%, 90%, and 95%. After converting these percentages to decimal form, we proceed this way:

$$0.70 \times 0.90 \times 0.95 = 0.5985$$
$$100\% = 1.00$$
$$1.00 - 0.5985 = 0.4015$$
$$0.4015 \times 720 = 289.08$$
$$720 - 289.08 = 430.92$$

We've determined that the discount is $289.08, leaving a selling price of $430.92, which agrees with our first solution.

Checking the Selling Price

The person filling out a bill or invoice must show the amount of discount, but the one who receives the bill just wants to check the selling price. There's an easy way of doing this. It's very similar to the short method of finding a single discount equal to a series of discounts.

Proceed just as you would if you were going to reduce the given discount rates to a single rate, but don't subtract the product of the remainders from 100%. Instead, multiply the product of the remainders by the list price. The final product will be the selling price. Thus, applying this method to our first example, your only two steps would be these:

$$.60 \times 0.90 \times 0.95 = 0.513$$
$$0.513 \times \$300 = \$153.90$$

Your checking procedure has determined that the selling price of $153.90 is correct.

In our second example, your steps would be

$$0.70 \times 0.90 \times 0.95 = 0.5985$$
$$0.5985 \times \$720 = \$430.92$$

Thus, you've checked and determined that the selling price of $430.92 is correct.

Cash Discounts

Besides the trade discounts, businesses frequently allow a discount for prompt payment. Such discounts are called *cash discounts.* An invoice for goods upon which such a discount applies will have a notation on it like this:

Terms: 3/10, n/30

This means 3%, 10 days; net 30 days. In other words, the net amount is due within 30 days, but a discount of 3% is allowed if the bill is paid within 10 days (Figure 12).

INVOICE

Chicago, Illinois
January 3, 19—

Mr. Walter R. Speck
Boston, Massachusetts

Purchased from: Cohen & Company
518 Broad Street
New York City, NY 10004

Terms: 3/10, n/30

16	Dozen white towels @$17.50	280.00

FIGURE 12—The terms shown on this invoice show you that you have 10 days to receive a 3% discount. The net amount is due within 30 days.

Sometimes an invoice will show a notation like this:

Terms: 3/10, 1/30, n/60

This means that you may deduct 3% if you pay within 10 days. If you pay after 10 days but before 30 days, you may deduct 1%. If you make payment after 30 days, you must pay the net amount before 60 days have expired.

Naturally, the *amount* of a cash discount can't be shown on an invoice because the seller doesn't know when you'll pay the bill. For this reason, you must compute whatever cash discount is due you on the invoice cost when you pay the bill.

Shipping Charges

Another thing that affects the net cost of goods to the buyer is the cost of shipping the merchandise. Who pays this charge is a very important question. You've probably seen or heard the abbreviation *F.O.B.,* which stands for "free on board." It means that the shipper will pay transportation charges to the place named. For example, if a Philadelphia concern sells goods "F.O.B. Philadelphia," they mean they'll deliver the goods to the freight yard in Philadelphia. The buyer will have to pay the charges from that point. If this same firm were to sell goods "F.O.B. destination," it would pay all the freight charges to the buyer's city (Figure 13).

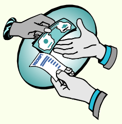
FIGURE 13—Always check the F.O.B. terms of purchase. The transportation charges can be substantial.

Now you should be able to solve problems that involve trade discounts, cash discounts, and shipping charges. Let's try an example problem.

Suppose you purchase a set of dinnerware, listed in the catalog at \$300, F.O.B. factory; less 30%, 10%; terms 3/10; freight \$25. If you pay for the dinnerware within a week, how much will your total cost be? Your calculations would look like this:

Single discount is	$0.70 \times 0.90 = 0.63$ $1.00 - 0.63 = 0.37$ $\$300 \times 0.37 = \$111$
Discount for paying cash in 10 days is	$(\$300 - \$111) \times 3\% =$ $\$189 \times .03 = \$5.67$
Your total cost is	$\$300 - \$111 = \$189$ $\$189.00 - \$5.67 = \$183.33$ $\$183.33 + \$25$ freight $= \$208.33$

Before reviewing the material in this study unit and taking the examination, take time to complete *Count Your Change 9*.



Count Your Change 9

A. Calculate the discount and selling price on each of the following.

	Marked Price	Discount	Amount of Discount	Selling Price
1.	\$100	12%	______	______
2.	\$75	20%	______	______
3.	\$60	25%	______	______
4.	\$72.80	15%	______	______
5.	\$150	8%	______	______
6.	\$980	$12\frac{1}{2}\%$	______	______
7.	\$24.98	$16\frac{2}{3}\%$	______	______
8.	\$47.70	$33\frac{1}{3}\%$	______	______

B. Find the selling price if the list price is \$400 and the trade discount is as follows.

1. 40% ______
2. 20% ______
3. 25% ______
4. $33\frac{1}{3}\%$ ______
5. $37\frac{1}{2}\%$ ______
6. $16\frac{2}{3}\%$ ______
7. 50% ______
8. 5% ______
9. $12\frac{1}{2}\%$ ______
10. 55% ______
11. 60% ______
12. $27\frac{1}{2}\%$ ______

C. Express the following as a single discount rate.

1. 25% and 16% ______
2. 30%, 20%, and 5% ______
3. 60%, 10%, and 5% ______
4. 40%, 20%, $12\frac{1}{2}\%$, and 4% ______
5. 25%, 20%, and 3% ______
6. $33\frac{1}{3}\%$, 25%, and 5% ______
7. What is the selling price of an article that lists at \$1500 subject to discounts of 40%, 25%, and 10%? (You may use the method described in “Check Your Selling Price.”)

   ____

8. What is the discount on \$640 if the discount rates are 25%, 10%, and 5%?

   ____

(Continued)

Count Your Change 9

D. Find the lowest price you would have to pay for each of the following.

1. Radio \$500, F.O.B. factory; less 30%, 10%, 5%; terms 2/10; freight \$62

   ____

2. Office machine \$1200, F.O.B. factory; less 20%, 10%, 10%; terms n/30; freight \$75

   ____

3. A businessman in Philadelphia wants to buy electric heaters. His best sources are two firms in Chicago. Let's call them Firm A and Firm B. Firm A lists a heater at \$291 less $33\frac{1}{3}\%$ and 10%, F.O.B. Chicago. Firm B lists the same heater at \$350, less 30% and 20%, F.O.B. destination. In addition, both firms offer a 2% discount for cash within 10 days. The freight on each heater would be \$22.75. What will be the least cost of the heater from each concern?

   Firm A ______________________________

   Firm B ______________________________

4. You've bought 20 calculators listed at \$125 each less 15%, 10%, and 5%. The date of the invoice was June 28, and you paid the bill on July 6. What amount did you pay if the terms were 3/10, n/60?

   ____

5. A trailer is selling for \$1150 less $33\frac{1}{3}\%$, 10%, and 4%. What is the invoice cost?

   ____

Check your answers with those on page 54.

Count Your Change Answers

1

1. True
2. False
3. False
4. True

2

A.

1. 3, 15, 1, 15, 11, 6, 4, 2, 9, 7
2. 7, 13, 8, 16, 11, 10, 10, 7, 4, 11
3. 9, 6, 5, 11, 12, 10, 12, 14, 13, 2
4. 6, 4, 9, 10, 6, 8, 8, 3, 12, 14
5. 14, 11, 13, 13, 8, 7, 12, 8, 13, 9
6. 3, 12, 18, 5, 11, 9, 5, 6, 12, 7
7. 11, 3, 14, 10, 6, 6, 0, 9, 12, 15
8. 2, 15, 7, 8, 13, 11, 5, 9, 7, 9
9. 17, 7, 16, 8, 9, 10, 8, 10, 16, 1
10. 10, 9, 10, 8, 14, 4, 5, 17, 6, 4

B.

1. 376
2. 441
3. 4551
4. 25,769
5. 430,857

3

1. 0, 1, 9, 9, 1, 3, 5, 8, 2, 9
2. 9, 1, 9, 6, 0, 7, 0, 8, 7, 4
3. 3, 5, 8, 9, 5, 8, 8, 6, 2, 7
4. 0, 4, 2, 7, 4, 2, 4, 8, 7, 0
5. 8, 0, 7, 3, 1, 8, 5, 4, 0, 5
6. 7, 3, 4, 0, 3, 9, 1, 5, 0, 6
7. 7, 1, 9, 4, 6, 6, 6, 0, 5, 2
8. 3, 8, 7, 3, 4, 5, 5, 7, 3, 3
9. 3, 1, 5, 8, 6, 9, 6, 2, 2, 2
10. 1, 4, 1, 2, 2, 6, 6, 1, 9, 4

4

1. 2, 56, 0, 56, 18, 0, 3, 0, 20, 12
2. 12, 42, 7, 64, 18, 25, 24, 0, 0, 28
3. 14, 8, 6, 24, 35, 9, 27, 48, 40, 1
4. 0, 3, 18, 16, 8, 7, 0, 0, 36, 48
5. 49, 28, 40, 42, 15, 6, 27, 16, 36, 8
6. 0, 32, 81, 4, 30, 0, 0, 9, 32, 10
7. 30, 2, 45, 16, 5, 0, 0, 14, 35, 18
8. 0, 54, 0, 12, 36, 24, 4, 8, 10, 18
9. 72, 6, 63, 12, 0, 21, 0, 24, 63, 0
10. 21, 20, 9, 15, 45, 0, 6, 72, 5, 4

5

1.
$$\begin{array}{r} 2694r20 \\ 23\overline{)61982} \\ -46 \\ \hline 159 \\ -138 \\ \hline 218 \\ -207 \\ \hline 112 \\ -92 \\ \hline 20 \end{array}$$

2.
$$\begin{array}{r} 21r26 \\ 37\overline{)803} \\ -74 \\ \hline 63 \\ -37 \\ \hline 26 \end{array}$$

3.
$$\begin{array}{r} 29r4 \\ 28\overline{)816} \\ -56 \\ \hline 256 \\ -252 \\ \hline 4 \end{array}$$

4.
$$\begin{array}{r} 4r31 \\ 47\overline{)219} \\ -188 \\ \hline 31 \end{array}$$

5.
$$\begin{array}{r} 32 \\ 82\overline{)2624} \\ -246 \\ \hline 164 \\ -164 \\ \hline 0 \end{array}$$

6.
$$\begin{array}{r} 694r32 \\ 72\overline{)5000} \\ -432 \\ \hline 680 \\ -648 \\ \hline 320 \\ -288 \\ \hline 20 \end{array}$$

7.
$$\begin{array}{r} 31 \\ 43\overline{)1333} \\ -129 \\ \hline 43 \\ -43 \\ \hline 0 \end{array}$$

8.
$$\begin{array}{r} 408 \\ 16\overline{)6528} \\ -64 \\ \hline 128 \\ -128 \\ \hline \end{array}$$

6

A.

1. 3.2
2. 8.79
3. 20.89
4. 12.564
5. 16.57
6. 10.11
7. 68.26

B.

1. 107.40
2. 403.52
3. 237.79

C.

1. 0.47531
2. 98.7252
3. 13,238.984
4. 410.688
5. 11,045.411

D.

1. 0.00025
2. 15,000
3. 0.175
4. 0.00983

E.

1. $\frac{5}{6}$
2. $\frac{15}{24}$, or $\frac{5}{8}$
3. $\frac{29}{44}$
4. $\frac{51}{105}$ or $\frac{17}{35}$
5. $\frac{59}{60}$

F.

1. $\frac{3}{8}$
2. $\frac{2}{15}$
3. $\frac{7}{12}$
4. $\frac{4}{17}$

G.

1. $\frac{1}{3}$
2. $1\frac{1}{8}$
3. $1\frac{1}{6}$
4. $\frac{3}{10}$
5. $4\frac{2}{3}$
6. $1\frac{4}{7}$

H.

1. 6
2. $1\frac{1}{3}$
3. $2\frac{1}{2}$
4. $1\frac{1}{20}$

7

A.

1. 345,000
2. 128,000
3. 675,000
4. 356,000

B.

1. 42.5
2. 39.6
3. 26.4
4. 47.5

C.

1. \$365.47, \$365
2. \$125.65, \$126
3. \$62.76, \$63
4. \$38.42, \$38

8

A.

1. $\frac{25}{100} = \frac{1}{4}$ 0.25
2. $\frac{32\frac{1}{2}}{100} = \frac{\frac{65}{2}}{100} = \frac{65}{200} = \frac{13}{40}$ 0.325
3. $\frac{4}{100} = \frac{1}{25}$ 0.04
4. $\frac{75}{100} = \frac{3}{4}$ 0.75
5. $\frac{6\frac{1}{2}}{100} = \frac{\frac{13}{2}}{100} = \frac{13}{200}$ 0.065
6. $\frac{125}{100} = \frac{5}{4}$ 1.25
7. $\frac{12\frac{1}{2}}{100} = \frac{\frac{25}{2}}{100} = \frac{25}{200} = \frac{1}{8}$ 0.125
8. $\frac{0.2}{100} = \frac{2}{1000} = \frac{1}{500}$ 0.002
9. $\frac{0.75}{100} = \frac{75}{10{,}000} = \frac{3}{400}$ 0.0075
10. $\frac{107}{100}$ 1.07
11. $\frac{210}{100} = \frac{21}{10}$ 2.10
12. $\frac{22\frac{1}{2}}{100} = \frac{\frac{45}{2}}{100} = \frac{45}{200} = \frac{9}{40}$ 0.225

B.

1. 0.4, or 40%
2. 3.25, or 325%
3. 0.3, or 30%
4. 0.375, or 37.5%
5. 7.26, or 726%
6. 0.125, or 12.5%
7. 0.31, or 31%
8. 1.75, or 175%
9. 22.33, or 2233%
10. 2.875, or 287.5%
11. 0.875, or 87.5%
12. 0.00125, or 0.125%

C.

1. $800 \times 0.25 = 200$
2. $70 \times 0.45 = 31.5$
3. $120 \times 0.333 = 39.96$, or 40
4. $\$600 \times 0.055 = \$33$
5. 580 bu × 1.25 = 725 bu
6. 740 cases × .0250 = 18.5 cases
7. 2764 ft × .0025 = 6.91 ft
8. 6784 lb × .0425 = 288.32 lb
9. 972 in. × .20 = 194.4 in.
10. 96 min × .375 = 36 min
11. 720 tons × .6666 = 479.95 or 480 tons
12. $450 \times .035 = 15.75$

D.

1. $\frac{12}{72} = 0.167$, or 16.7%
2. $\frac{28}{224} = 0.125$, or 12.5%
3. $\frac{40}{20} = 2$, or 200%
4. $\frac{99}{44} = 2.25$, or 225%
5. $\frac{126}{8400} = 0.015$, or 1.5%

E.

1. $\frac{50}{0.20} = 250$
2. $\frac{18}{0.333} = 54$ (rounded)
3. $\frac{4}{0.01} = 400$
4. $\frac{90}{0.05} = 1800$
5. $\frac{72}{.1667} = 431.92$

F.

1. $30 - 20 = 10$; $\frac{10}{30} = 0.33\frac{1}{3}$, or $33\frac{1}{3}\%$
2. $25 - 20 = 5$; $\frac{5}{20} = 0.25$, or 25%
3. $\frac{37-32}{32} = 0.156$, or 15.6%
4. $\frac{599-300}{599} = 0.499$, or 49.9%
5. $\frac{184-148}{148} = 0.243$, or 24.3%
6. $\frac{102-50}{50} = 1.04 = 104\%$
7. $\frac{489-16}{489} = .967 = 96.7\%$
8. $\frac{216-80}{80} = 1.7 = 170\%$
9. $\frac{516-418}{516} = .1899 = 18.99\%$
10. $\frac{5}{30} = 0.1667$, or 16.7%
11. $\frac{65-60}{65} = 0.077$, or 7.7%
12. $\frac{196-132}{196} = 0.327$, or 32.7%
13. $\frac{21{,}840-17{,}190}{21{,}840} = 0.213$, or 21.3%
14. $\frac{184{,}211-157{,}928}{157{,}928} = 0.166$, or 16.6%

9

A.	Discount	Selling Price
1.	$\$100 \times 0.12 = \$12.00$	$\$100 - \$12 = \$88$
2.	$\$75 \times 0.20 = \$15.00$	$\$75 - \$15 = \$60$
3.	$\$60 \times 0.25 = \$15.00$	$\$60 - \$15 = \$45$
4.	$\$72.80 \times 0.15 = \$10.92$	$\$72.80 - \$10.92 =$ $\$61.88$
5.	$\$150 \times 0.08 = \$12.00$	$\$150 - \$12 = \$138$
6.	$\$980 \times 0.125 = \$122.50$	$\$980 - \$122.50 =$ $\$857.50$
7.	$\$24.98 \times 0.1667 = \$4.16$	$\$24.98 - \$4.16 =$ $\$20.82$
8.	$\$47.70 \times 0.3333 = \$15.90$	$\$47.70 - \$15.90 =$ $\$31.80$

B.

1. $\$400 \times 0.6 = \$240$
2. $\$400 \times 0.80 = \$320$
3. $\$400 \times 0.75 = \$300$
4. $\$400 \times .6667 = \$266.68$
5. $\$400 \times 0.625 = \$250$
6. $\$400 \times .8333 = \$333.32$
7. $\$400 \times 0.50 = \$200$
8. $\$400 \times 0.95 = \$380$
9. $\$400 \times 0.875 = \$350$
10. $\$400 \times 0.45 = \$180$
11. $\$400 \times 0.40 = \$160$
12. $\$400 \times 0.725 = \$290$

C.

1. $0.75 \times 0.84 = 0.63$
   $1.00 - 0.63 = 0.37$, or 37%

2. $0.70 \times 0.80 \times .95 = 0.532$
   $1.000 - 0.532 = 0.468$, or 46.8%

3. $0.40 \times 0.90 \times 0.95 = 0.342$
   $1.000 - 0.342 = 0.658$, or 65.8%

4. $0.60 \times 0.80 \times 0.875 \times 0.96 = 0.4032$
   $1.000 - 0.4032 = 0.5968$, or 59.68%

5. $0.75 \times 0.80 \times 0.97 = 0.582$
   $1.000 - 0.582 = 0.418$, or 41.8%

6. $.667 \times 0.75 \times 0.95 = 0.475$
   $1.000 - 0.475 = 0.525$, or 52.5%

7. $0.60 \times 0.75 \times 0.90 = 0.405$
   $\$1500\ \big)0.405 = \$607.50$

8. $0.75 \times 0.90 \times 0.95 = 0.64125$
   $1.00000 - 0.64125 = 0.35875$
   $\$640 \times 0.35875 = \$229.60$

D.

1. $0.70 \times 0.90 \times 0.95 = 0.5985$
   $1.0000 - 0.5985 = 0.4015$
   $500 \times 0.4015 = 200.75$ (Discount)
   $(500 - 200.75) \times 0.02 = 299.25 \times 0.02 = 5.985$,
   or 5.99 (Cash discount)
   $299.25 - 5.99 + 62.00 = \$355.26$

2. $0.80 \times 0.90 \times 0.90 = 0.648$
   $1.0000 - 0.648 = 0.352$
   $1200 \times 0.352 = 422.40$
   $(1200 - 422.40) + 75.00 = \$852.60$

3. . Firm A:
   $.6667 \times 0.90 = 0.60$
   $1.00 - 0.60 = 0.40$
   $291 \times 0.40 = 116.40$
   $(291 - 116.40) \times 0.02 =$
   $174.60 \times 0.02 = 3.49$
   $(174.60 - 3.49) + 22.75 = \$193.86$

   . . . Firm B:
   $0.70 \times 0.80 = 0.56$
   $1.00 - 0.56 = 0.44$
   $350 \times 0.44 = 154.00$
   $(350 - 154) \times 0.02 = 3.92$
   $196.00 - 3.92 = \$192.08$
   (Note that Firm B pays freight charges to destination.)

4. $125 \times 20 = 2500$
   $0.85 \times 0.90 \times 0.95 = 0.72675$
   $(1.00000 - 0.72675) \times 2500 = 683.13$
   $(2500 - 683.13)\ \big)0.03 = 54.51$
   $1816.87 - 54.51 = \$1762.36$

5. $1.000 - (.6667 \times 0.90 \times 0.96) = 0.424$
   $1150.00 - (1150 \times 0.424) = \$662.40$

Glossary

Add To combine or unite in a group. For example, a group of seven elements can be combined with a group of eleven elements to form a group of eighteen elements.

Additive That which is added. An additive expression is one in which all the numbers are to be added.

Base In figuring percents and percentages, the base is the number by which a percentage is figured. The *percentage* times the *rate* equals the *base.*

Cancellation In multiplying fractions, cancellation is the practice of dividing any numerator and any denominator by the same number. In the example below, the 3 and the 9 have both been divided by 3, and the 5 and the 20 have been divided by 5. The answer is the *product* of the *quotients.*

$$\frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{20}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{9}}} = \frac{\overset{1}{\cancel{1}}}{\underset{3}{\cancel{12}}}$$

Decimal A decimal number is a fraction that has a power of ten as its denominator. Because it's in base ten, it need not be expressed as a fraction usually is, with a numerator and denominator, but can be represented by a place value. A decimal point is the dot written between two digits to separate the fractional part of a number from the rest of that number.

Denominator The denominator is the number below the line in a fraction. The denominator names, or *denominates,* the number of equal parts into which the whole will be divided. In the fraction $\frac{1}{2}$, 2 is the denominator. All fractional numbers are actually divisions, so a denominator is very much the same thing as a divisor.

Difference The answer to a subtraction problem.

Discount A reduction of value, price, or cost, expressed as a percent of a number. Therefore, if you're offered a 10% discount on a coat that costs $210, you'll be able to take $21 off the price and buy it for $189.

Divide To split a group into equal parts. When dividing, you'll be asked to find either the size of each part, or the number of parts of a certain size within the group. For example, if you're asked to divide 420 into 7 equal groups, you would discover that each group of 7 had 60 members. Or if you were asked how many groups containing 60 members could be taken from 420, you would find that there are 7 such groups, since $420 \div 60 = 7$.

Dividend The number that's to be divided in a division problem. In the problem $100 \div 10 = 10$, 100 is the dividend.

Division The process of dividing, or splitting groups into equal parts. Dividing is the opposite of multiplying.

Divisor The number by which you're dividing another. In $36 \div 6$, 36 is the dividend, 6 the divisor.

Factor A number that's multiplied to make a product. 3 and 4 are factors of 12; so are 2 and 6, and 1 and 12.

Fraction A fraction is a part of a whole. It represents a division, worked vertically instead of horizontally. It's usually written with one number (the numerator) above another number (the denominator), separated by a horizontal line. The numerator corresponds to a dividend, and the denominator, below, to the divisor.

Gross The total amount. In terms of money or paychecks, the gross is the amount you make before deductions; the net is what you take home. Or if you were shipping three pounds of apples packed in $\frac{1}{2}$ pound of packing, wrapped in a $\frac{1}{4}$ pound box, the gross weight of the package would be $\frac{3}{4}$ pounds.

Least Common Denominator In adding and subtracting fractions, the least common denominator (LCD) is the smallest number of which all the denominators are factors. Therefore, in $\frac{1}{5} - \frac{4}{20}$, 20 is the LCD, since both 5 and 20 go into it evenly. In $\frac{1}{4} + \frac{2}{3}$, 12 is the least common denominator.

Minuend In subtraction, the minuend is the number from which another (the *subtrahend*) is subtracted. In the problem $11 - 7 = 4$, 11 is the minuend, 7 the subtrahend, and 4 is the difference.

Multiplicand The number that's being multiplied. In 28×5, 28 is the multiplicand. In vertical multiplication problems, the multiplicand is the top number.

Multiplier The second factor of a multiplication problem. It multiplies the multiplicand.

Multiply To add groups of a set size a given number of times. For example, 4×7 is the same as writing $7 + 7 + 7 + 7$, only faster.

Net An amount from which deductions have been made. A box of corn flakes may have a gross (total) weight of 12 oz and a net weight of $11\frac{1}{2}$ oz—meaning you're really getting $11\frac{1}{2}$ oz of corn flakes.

Numeral The symbol used to denote a number. For example, 3 and III are both numerals for the concept *three*.

Numerator In fractions, the number that's written above the dividing line. If you think of fractions as division, the numerator is the *dividend*. In $\frac{3}{4}$, 3 is the numerator, 4 the denominator.

Percent (%) A fractional number with a denominator of 100. 3% means $\frac{3}{100}$, or .03. Three percent of a given number means $\frac{3}{100}$ of that number.

Percentage A part of a whole. In figuring 30% of $120 = 36$, 36 is the percentage.

Prime number A prime number is a natural number (that is, any whole number—not a fraction or a negative number except zero), which can be evenly divided only by itself and 1. The numbers 1, 2, 3, 5, and 7 are all prime numbers.

Product The number you get by multiplying any two numbers. In $7 \times 3 = 21$, the product is 21.

Quotient The number you get by dividing one number by another—that is, the answer to a division problem. In $54 \div 9 = 6$, 6 is the quotient.

Rate In figuring percents, rate is the number which, when multiplied by the *base,* yields the percentage.

Remainder The undivided part of a division problem. For example, in $23 \div 5$, 5 goes into 23 four times, with a remainder (undivided part) of 3.

Rounding off To show a number only to a given degree of accuracy—that is, to approximate a number accurately. For example, if we were rounding off 3.14156 to the nearest whole number, it would be 3. If we were rounding it off to the nearest *tenth*, we would round it to 3.1. But if we were rounding it to the nearest *ten-thousandth*, we would round it to 3.1416, since for any number greater than or equal to five, we round *up*. If we were rounding to the nearest whole number, 4.999 would be rounded up to 5.0

Subtotal A total midway through an operation; a sum which will be added to other sums to form a final sum, the total.

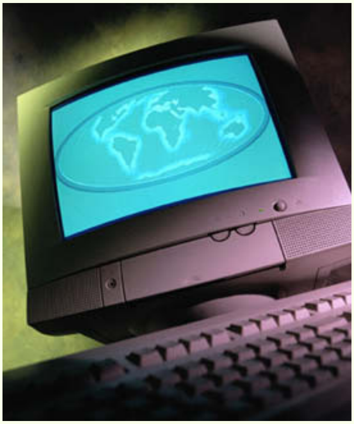
Subtract To remove, or take away, numbers from a group. Subtraction is the inverse of addition.

Subtrahend The number that's to be subtracted from another. In the problem $16 - 4 = 12$, 4 is the subtrahend.

Sum The answer to an addition problem. In $15 + 3 = 18$, 18 is the sum.

Total The sum that results from adding sums. The sums that are added to form the total are called *subtotals*.

Whole number Any of the natural numbers—that is, a positive number that can be expressed without a fraction—1, 2, 3, 4, 5, 6, etc., but including zero. The set of whole numbers is 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, etc.



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